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Filling flows, cliff erosion and cleaning flows

By D. H. PEREGRINE AND S. KALLIADASIS†

School of Mathematics, University Walk, Bristol BS8 1TW, UK

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The flows considered here are those where a container or confined region is being filled by a substantial flow of liquid. The case of especial interest is where the incoming flow fills a large part of the cross-section of the container, for example where a nearly full flowing conduit has one end suddenly closed and hence fills rapidly, or where a water wave propagates close to the under surface of a horizontal structure and then rapidly fills the available space. These flows are taken to be so rapid that gravity is unimportant and yet not so violent that compressibility effects become significant. Important features, such as the greatly enhanced pressures and a thin high-velocity return jet are evaluated. The calculated pressures are very significantly greater than those associated with the incoming flow velocity and can be especially large when there is little clearance between the flow and the container boundary. One of many possible applications is in the extension of cracks and openings in coastal cliffs and structures. The flows could also be relevant to estimating the forces on the underside of some marine structures. A simple two-dimensional irrotational free-surface solution is found for the flow, which is steady in a suitably moving frame of reference.

Reversing the direction of one of these filling flows gives the case of a narrow high-speed jet which may be used to flush liquid out of cavities and containers. The optimum size of jet is calculated.

1. Introduction

This study is motivated by consideration of the violent motions that may ensue when a water wave impinges into a confined space on a cliff or a coastal structure. For simplicity a horizontal slot is considered in much of the discussion. If the solid surfaces are reasonably smooth a stream of water flows in, hits the far end and fills the available space. If this flow is relatively fast, jet like, and thin enough it can meet the far end, be turned around and shoot back out along the upper surface of the space. A similar flow is sometimes seen in the kitchen when full flow from a tap falls into an empty cup and is simply returned out of the cup when no dissipative breaking of the free surface occurs. We shall discuss the filling flow as being horizontal and any return flow as being along the top of the confining space although the flow configuration can be at any angle since gravity is ignored. Here we are particularly concerned with the case where there is no dissipation but the incoming flow fills so much of the space available that most of the flow contributes to filling whilst a smaller proportion returns along the top of the available space. We show that for small clearances the return flow is very small so that it may be reasonable to ignore it in many applications, in the same way as the small return jet in front of a hydrofoil is often ignored in theoretical studies.

† Present address: Department of Chemical Engineering, Univeristy of Leeds, Leeds LS2 9JT, UK.

There are some related flow problems. At one extreme there is the case of flow which completely fills a pipe. If this flow is suddenly stopped by a valve or other mechanism, the resulting pressure surge, or water hammer, leads to very high transient pressures that depend on the compressibility of the filling liquid and the elasticity of the containing walls. The major development of this topic was by Joukowski (1900) and it is now well covered in various textbooks. In this context the case we consider corresponds to a nearly horizontal pipe which is partly full of rapidly moving liquid which is suddenly stopped at a valve: the flow then becomes a filling flow.

There is one two-dimensional solution including gravity which is due to Benjamin (1968). This is derived as an 'emptying' flow for the liquid between two horizontal planes, but, being inviscid, it can run in either direction and hence be interpreted as a filling flow. The free surface rises to the upper boundary, approaching it at an angle of $\pi/3$ to the horizontal. However, this is appropriate only for one single inflow/outflow velocity with Froude number $\surd 2$, and the depth of liquid is just half the distance between the two horizontal bounding planes. The flows described below can be considered as a gravity-free, overturning, version of the Benjamin flow. That is these flows are dominated by the liquid's inertia.

The primary idea for this work comes from studies of wave impact (Cooker & Peregrine 1990*a*,*b*, 1992) where it is found that very severe wave forces can occur when a nearly breaking wave meets a wall without any actual breaking or impact. The water at the wall accelerates violently upwards just in front of the near-vertical wave face. This motion we call 'flip through'. It has a maximum pressure shortly after the formation of a thin jet directed up the wall. These flows are fully described by an inviscid irrotational flow and Cooker (private communication) has noticed that near the time of maximum pressure the small region in which high pressure occurs is well modelled by a steady jet-like flow in a suitable moving reference frame (Cooker & Peregrine, in preparation).

The analysis below describes the filling of the space between two horizontal planes, but has wider applications. For example, the flow may be used to describe the latter stages of filling of a container of any shape when the region of the fill level has cylindrical geometry; this might be extended to cases with gentle variations of the container's geometry or slow variations in the filling flow. Further, the far end of the horizontal space may be open and it could be filled from both ends with similar flows; this may be more appropriate for considering pressures on the underside of the decks of some marine structures.

The next section describes the relationship between the unsteady filling flow and a steady flow in a moving reference frame. Consideration of the conserved quantities gives equations from which the main properties of the flow are readily found. These include the pressures that develop in the main body of the container and the peak pressure. In the steady flow this is at the stagnation point, and hence easy to evaluate.

Section 3 discusses the filling flow in the context of coastal wave impacts. The following section gives a brief discussion of cleaning flows which correspond to the same flow solution but with all velocities reversed.

2. Transformation to a steady flow

The container is described as the space between two horizontal planes distance H apart. This orientation is chosen for convenience of description since the flow is considered to be sufficiently fast that gravitational effects are negligible. Note, as may be seen from the results, that this does not necessarily imply that the incoming flow,

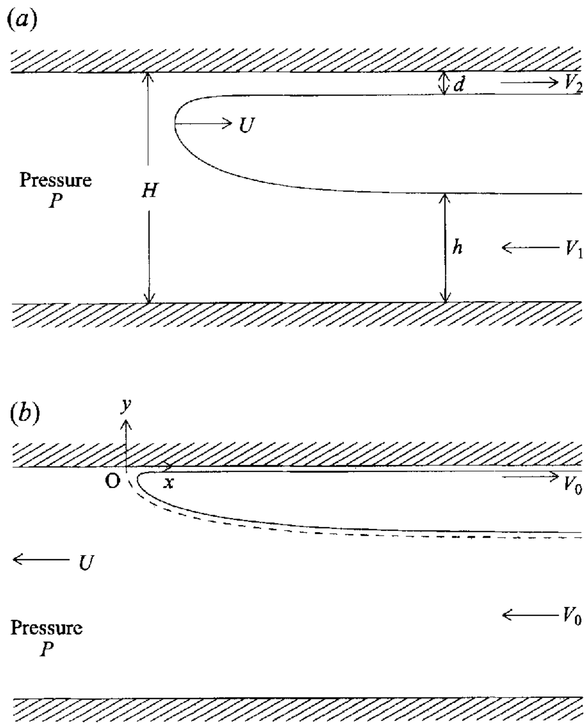

FIGURE 1. The configuration of the filling flow: (*a*) on the stationary frame, for $h = 0.468H, d = 0.1H$; (*b*) in the moving reference frame where the flow is steady including the dividing streamline - - - - -, for $h = 0.709H, d = 0.025H$.

of depth h and velocity V_1, necessarily has a high Froude number, $V_1/(gh)^{1/2}$, in the conventional sense, if the flow is nearly horizontal. Precise requirements depend more on the space over the filling flow and its orientation relative to gravity, as is discussed later in this section. Figure 1(*a*) shows the configuration and the notation used. The thinner backflow has velocity V_2 and thickness d. The free surface is assumed to have a constant form and to be moving, as the space is filled, at a velocity U.

Now, consider the flow in a reference frame moving with the free surface; figure 1(*b*) shows the new flow. On the free surface Bernoulli's equation gives that the velocity is constant, V_0, say. The velocity in distant parts that are already filled is no longer zero but is U. There must be a stagnation point on the upper rigid surface where the flow divides.

The relative velocities between the two frames give us

$$V_0 = V_2 - U = V_1 + U. \tag{1}$$

Mass conservation, choosing units in which the liquid density is unity, gives

$$UH = V_0(h - d). \tag{2}$$

Consideration of the momentum flux gives:

$$U^2H + PH = V_0^2h + V_0^2d, \tag{3}$$

where P is the excess pressure in the filled portion of the space. (Note that using Benjamin's terminology of 'flow force' too loosely, can lead to an error of sign in the right-hand side of this equation since the momentum flux is a tensor and does not have the same character of direction as a vector, for example compare with tension in a string which is another one-dimensional tensor.) Using Bernoulli's equation gives:

$$P = \tfrac{1}{2}(V_0^2 - U^2). \tag{4}$$

The natural specification of this problem is for H, h and V_1 to be given. It is then straightforward to deduce the other quantities from equations (1) to (4) with the results

$$U = \tfrac{1}{2}V_1(2k-1)/(1-k), \tag{5}$$

$$V_0 = \tfrac{1}{2}V_1/(1-k), \tag{6}$$

$$V_2 = V_1 k/(1-k), \tag{7}$$

$$d = H(1-k)^2, \tag{8}$$

$$P = \tfrac{1}{2}V_1^2 k/(1-k), \tag{9}$$

$$P_0 = \tfrac{1}{8}V_1^2/(1-k)^2, \tag{10}$$

where P_0 is the maximum pressure at the 'stagnation point', and $k = (h/H)^{1/2}$. Since units can be chosen with H and V_1 equal to unity, k, or h/H, is the single dimensionless parameter on which these flows depend.

Note the occurrence of $(1-k)$; when it becomes small, all these velocities and pressures become large, whilst d becomes very small. For example, for an inflow depth of $h = 0.81H$, the return flow has thickness $d = 0.01H$, and return velocity $V_2 = 9V_1$, the internal pressure is $P = 9(\frac{1}{2}V_1^2)$, and the maximum pressure is $P_0 = 25(\frac{1}{2}V_1^2)$.

The necessary conditions on flow parameters for the filling flow to be applicable for a reasonable time can now be assessed, for example, when the flow takes place in a finite-gravity environment with a component of gravity across the container such that the thin jet will detach from the upper wall. If the entering flow is horizontal, the trajectory of the motion is a parabola which in dimensionless coordinates is given by

$$y = -\frac{x^2(1-k)^2}{2Fr^2 k^2}$$

where x and y are scaled with by H. The Froude number Fr is defined as $Fr^2 = V_1^2/gH$. We need $y < 1 - k^2$ to avoid interference with the incoming flow. The corresponding value of x is $Fr[2k(1+k)/(1-k)]^{1/2}$, which should be large for the filling flow solution to have a reasonable duration following the initiation of the jet. That is we seek

$$Fr^2 \gg \frac{(1-k)}{2k(1+k)}.$$

Therefore even with $Fr = O(1)$ our solution can be useful if $1-k$ is small enough. This criterion takes no account of air resistance which is significant if the jet breaks up into drops; but the general idea still holds. A somewhat different criterion is needed for a vertically oriented container, but for finite containers of moderate length the outflowing jet will often easily exit the container.

The steady two-dimensional inviscid flows are found from the standard free-streamline theory; details are given in the Appendix. For the steady version, corresponding to figure 1(b), with origin at the stagnation point, the complex velocity potential is:

$$f(z) = \phi(x,y) + \mathrm{i}\psi(x,y) = \frac{h}{\pi}\left\{2\log\zeta - (1-\zeta_0{}^2)\log\left(\frac{\zeta^2-\zeta_0{}^2}{1-\zeta_0{}^2}\right)\right\} \tag{11}$$

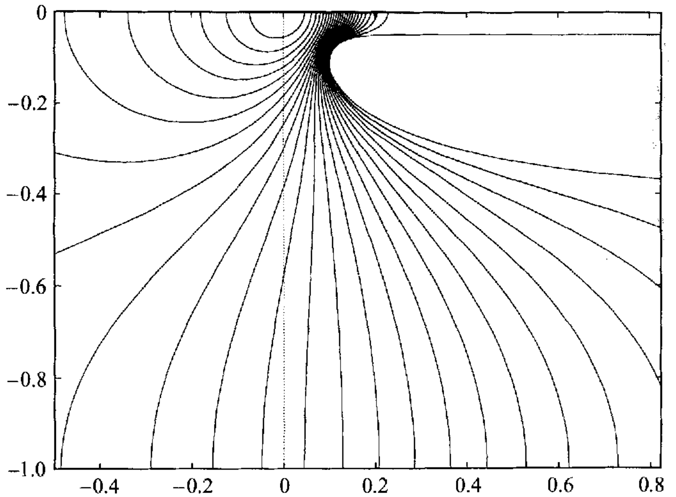

FIGURE 2. Pressure contours for $d/H = 0.05$, $P_0 = 2.5V_1^2$, contour interval: $0.1V_1^2$.

where for simplicity units have been chosen such that H and V_0 are both unity and where

$$\zeta = \frac{1+w}{1-w}, \quad {\zeta_0}^2 = \frac{d}{h} = \left(\frac{1-k}{k}\right)^2, \quad w = \frac{\mathrm{d}f}{\mathrm{d}z},$$

and

$$z = x + \mathrm{i}y = \frac{h}{\pi}\left\{-2\log\zeta + (1+\zeta_0)^2\log\left(\frac{\zeta-\zeta_0}{1-\zeta_0}\right) + (1-\zeta_0)^2\log\left(\frac{\zeta+\zeta_0}{1+\zeta_0}\right)\right\}. \quad (12)$$

The free streamline is given by $\zeta = -\mathrm{i}\lambda$ for $0 < \lambda < \infty$. Consideration of the physically relevant part of the ζ-plane shows that the logarithms should be chosen with a branch cut in the sector $\pi > \theta > \pi/2$. Figures 1(*a*) and 1(*b*) show the free surface for two examples. The pressure field for $d/H = 0.05$ is given in figure 2.

It should be noted that $d < h$ only for $h > \frac{1}{4}H$. For smaller values of h the return flow given by the above solution is of greater thickness. For filling flows this is probably an irrelevant solution, since for a two-dimensional containing space with a concave end e.g. a vertical wall or a rounded end, there are solutions corresponding to the filling flow failing to fill the container since it simply follows round the end of the container to its upper surface and proceeds towards the exit, if travelling fast enough to avoid falling etc.

Although the above analysis is for two-dimensional flows, the results (5)–(10) come from basic integral properties that are also applicable to three-dimensional flows such as occur in a pipe. The only change is that H, h and d should be interpreted as the cross-sectional areas of the pipe, the filling flow and the return flow respectively. The expressions for the velocities and pressures are unchanged, and k^2 becomes the ratio of the cross-sectional area of the exit flow to the area of the incoming flow. A three-dimensional flow is unlikely to be as tidy as the two-dimensional case since even if the filling flow has a smooth free surface, for example because of the action of gravity, the return flow is likely to be distinctly non-uniform because of three-dimensional motion near the 'stagnation point' leading to significant internal flows within the return jet. In a round pipe it can be expected to initially converge, and then diverge. Again, this solution is likely to be most useful when the return jet carries little liquid.

There is of course another filling flow for which there is no return flow, where all the incoming fluid contributes to the filling with turbulent dissipation, as in a hydraulic jump. The system of equations is now much simpler. Relative velocity and mass conservation now give

$$V_0 = V_1 + U, \quad V_0 h = UH, \quad \text{hence} \quad U = \frac{V_1 h}{H-h} = \frac{V_1 k^2}{1-k^2}, \tag{13}$$

so comparison with equation (5) shows that the filling is quicker, as is obviously the case, since there is no return flow. Momentum conservation gives

$$U^2 H + PH = V_0{}^2 h = \frac{U^2 H^2}{h}, \quad \text{and thus} \quad P = \frac{V_1{}^2 k^2}{(1-k^2)}. \tag{14}$$

This pressure is less than that of the non-dissipative flow, equation (9), for the appropriate range $\frac{1}{2} < k < 1$. If a filling flow occurred in too long a channel it may be expected to turn into a dissipative flow of this sort.

3. Cliff erosion

The filling flows described above can cause significantly elevated pressure in a crack or cavity during such time as the container is being filled through a section where the above type of flow can occur. Direct wave impact against a wall can also give exceptionally high pressures, both in the flip-through case already mentioned and when small air-pockets are trapped against a wall (work in progress). Thus a crack in a cliff or the structure of a breakwater can be exposed to pressures that may cause damage. The problem is to assess how dangerous these effects are. A feature of the very high pressure that direct wave impact can cause is that the duration of the high pressure decreases as its magnitude increases. For the peak pressures that are measured in the laboratory, typical durations are a few milliseconds, and a few hundredths of a second for coastal waves. There is the opportunity for significantly longer durations of high pressure from filling flows if a crack has appropriate length. Further, the very high pressures of direct impact only occur over a small area of the wall. Thus high filling pressures could be more common than high direct pressures at a given location. A worst case is for the high-speed jet from a flip-through to enter a crack and give even higher pressures.

The damage potential of these high pressures depends on the amount of work that they can do in loosening, fracturing or moving material. The simplest case could be where a slot underneath a block of stone fills with sufficient pressure at some instant to raise the block. The force on the surface of a crack can easily be estimated for a filling flow. The pressure in the filled portion of the crack, P, acts over a length that increases with speed U. If the crack length is L, then the maximum force is PL. If the total filling time, L/U, is short the total impulse,

$$P\frac{L^2}{2U} = V_1 L^2 \frac{k}{2(2k-1)}, \tag{15}$$

may be a more relevant quantity.

The above discussion ignores the extra pressure in the neighbourhood of the moving stagnation point. The analytic solution, (11) and (12), in the moving reference frame

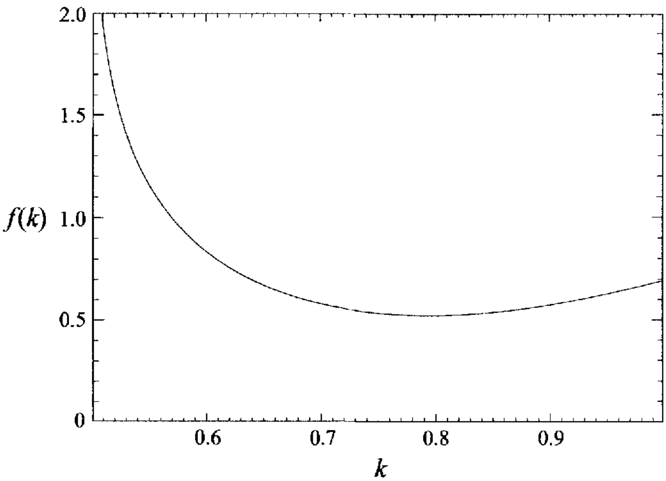

FIGURE 3. Plot of the stagnation-point force function $f(k)$.

permits description of this pressure by an 'extra' force, defined as

$$F_0 = \int_{-\infty}^{\infty} p\mathrm{d}x - P\int_{-\infty}^{0} \mathrm{d}x, \tag{16}$$

where this finite quantity is stated in a divergent manner in order to clarify its origin. The analytic solution gives

$$F_0 = \frac{V_1^2 H}{\pi}\frac{k}{1-k}f(k), \tag{17}$$

where

$$f(k) = (1-k)^2\log(1-k) + k^2\log k - \tfrac{1}{2}\log(2k-1) - (1-2k+2k^2)\log 2. \tag{18}$$

This function is shown in figure 3.

This force can be put in context as follows. The effective length over which the stagnation pressure P_0, acts is

$$\frac{F_0}{P_0} = \frac{8}{\pi}Hk(1-k)f(k) = \frac{8}{\pi}(hd)^{1/2}f(k). \tag{19}$$

Alternatively, the force corresponds to the filled pressure, P, acting over an extra length of

$$\frac{F_0}{P} = \frac{2H}{\pi}f(k). \tag{20}$$

A simple force balance indicates that the forces on the two bounding planes are equal and opposite despite the markedly differing pressure distributions.

Another case is where the space being filled has some air trapped at the closed end: a likely circumstance is sketched in figure 4. Work is done by the filling flow in compressing the air. If the air has relatively small volume then it will be compressed to the same filling pressure, P, as the water. If the air volume is relatively large so that its natural period of oscillation is similar to the filling time, the filling becomes unsteady since the pressure and volume of air at the end of the filling space are related. Thus the analysis of §2 must be amended. If the flow is considered to be quasi-steady a suitably amended version of equations (1)–(3) can be set up to allow an average velocity at the closed end of $W = (1/A)(\mathrm{d}G/\mathrm{d}t)$ where A is the cross-sectional

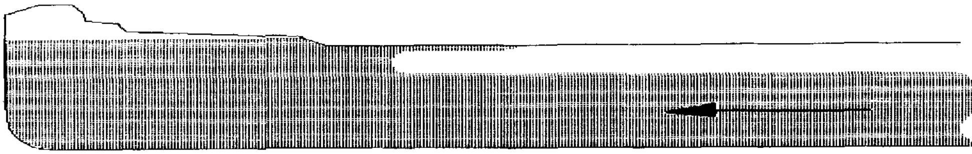

FIGURE 4. A filling flow with trapped air.

area of the filling passage. G is the trapped gas volume and decreases as the pressure P increases. Together with the pressure–volume relationship for the trapped gas this gives equations for the flow's evolution. Both the gas and water could permeate cracks in a cliff or marine structure. Clearly a filling flow can help to augment, or create, the jets of spray that can be seen from blow-holes at some coastal points.

It seems unlikely that this mechanism of creating high pressures would give much improvement in the useful energy from wave power devices, since to compress a significant volume of air to, say, one or two atmospheres above atmospheric would need relatively rapid filling velocity U along the filling portion which would not be consistent with large changes of volume in the trapped air.

4. Cleaning flows

Reversal of the flow direction in the above non-dissipative filling flow gives a high-speed jet clearing out a container. Typically, a high speed 'cleaning' jet is applied with no attempt to optimize the approach used since in most circumstances there is little need for optimization. However, should there be a need to use a cleaning jet to its best effect then the above results provide useful formulae. In this case we may be given the velocity V_2 of the jet, and, perhaps, can choose the amount, d, of the jet that is permitted to enter the container to be cleaned. Then equation (7) gives the velocity of efflux of liquid, including that which goes in. More usefully, the net rate of efflux of liquid from the container, in terms of V_2 and d is

$$V_1(h-d) = V_2 H \frac{m(1-2m)}{1-m}, \quad \text{where} \quad m^2 = \frac{d}{H}.$$

For given V_2 and H, this expression has a maximum when

$$m = 1 - \frac{1}{\sqrt{2}}, \quad \text{i.e.} \quad \frac{d}{H} = \frac{3}{2} - \sqrt{2} \approx 0.084;$$

thus the most effective size of jet to clean out a container has a cross-sectional area approximately equal to one twelth of the cross-section of the container.

5. Conclusion

The flows described here are likely to find useful application as simple models of a range of confined flows involving both gas and liquid. Flow in pipes of gas–liquid mixtures is widespread, in oil and gas extraction and conveyance, in steam raising for a multitude of purposes, and in large- and small-scale hydraulic works ranging from hydro-electric schemes to sewage disposal. Indeed it is not uncommon to see the basic conservation equations being used in hydraulic jumps and in their extensions in two-phase flows. The distinctive feature here is that we permit the high-speed return flow, and are specially interested in the high pressures that are generated. There is plenty of scope for experiments to compare with the results given here.

Both the simple integral solution and the detailed analytical solution can be used in various ways. For example, the case $k = 0.9$ detailed above can form an excellent basis for testing the accuracy of computer programs for unsteady irrotational free-surface flows.

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Appendix. The steady two-dimensional inviscid flow

The incoming flow in figure 1(*b*) is due to a source at $x = +\infty$ whilst the outgoing thin jet in the same figure is due to a sink at $x = +\infty$. A sink at $x = -\infty$ generates a flow with velocity U. The hodograph plane $w = u - iv$ can be easily constructed: the flow emanates from a source at $w = -1$ and disappears at the sinks located at $w = -(h-d)$ and $w = +1$ (for convenience we choose units such that H and V_0 are both unity). The lower half of the circle $|w| = 1$ is the free streamline. The transformation

$$\zeta = \xi + i\eta = \frac{1+w}{1-w}$$

maps the sink located at $w = +1$ of the (u, v)-plane to a sink at infinity in the ζ-plane, whilst the source at $w = -1$ is mapped to a source located at the origin of the ζ-plane. The point $w = -(h-d)$ of the hodograph plane is mapped to the point

$$\zeta_0 = \frac{1-(h-d)}{1+(h-d)}$$

of the ζ-plane. The negative imaginary ζ-axis is now the free streamline and the velocity potential can easily be found. It is the potential of a flow due to a source at $\zeta = 0$, a sink at $\zeta = \zeta_0$ and the image of this sink at $\zeta = -\zeta_0$:

$$f = \alpha \log \zeta - \beta \log(\zeta^2 - \zeta_0{}^2), \tag{A1}$$

where α and β are the strengths of the source and sinks respectively. Furthermore, the point $\zeta = 1$ must be a stagnation point in the ζ-plane also. Therefore,

$$\left(\frac{df}{d\zeta}\right)_{\zeta=1} = 0, \quad \text{or} \quad \alpha = \frac{2\beta}{1-\zeta_0{}^2}. \tag{A2}$$

Consideration of the flow out of the source of strength α and into the sink of strength β shows that one quarter the flow out of the source of strength α corresponds to the flow h, and half the flow into the sink of strength β corresponds to the flow $h - d$. Thus

$$\alpha = \frac{2\beta h}{h-d},$$

and equation (A2), with the definition of ζ_0 gives

$$(h-d)^2 - 2(h-d) + 1 - 4d = 0,$$

the solution of which is equation (8), as already found from the conservation of mass and momentum. These result in

$$\alpha = \frac{2h}{\pi}, \quad \text{and} \quad \beta = \frac{h-d}{\pi}.$$

Equation (A1) can now be differentiated with respect to ζ and with

$$\frac{\mathrm{d}f}{\mathrm{d}\zeta} = \frac{\mathrm{d}f}{\mathrm{d}z}\frac{\mathrm{d}z}{\mathrm{d}\zeta} = w\frac{\mathrm{d}z}{\mathrm{d}\zeta},$$

$$\frac{\mathrm{d}z}{\mathrm{d}\zeta} = \alpha\zeta_0{}^2 \frac{(\zeta+1)^2}{\zeta(\zeta-\zeta_0)(\zeta+\zeta_0)}, \tag{A3}$$

On integration,

$$\frac{z-c}{\alpha} = -\log\zeta + \tfrac{1}{2}(\zeta_0+1)^2\log(\zeta-\zeta_0) + \tfrac{1}{2}(\zeta_0-1)^2\log(\zeta+\zeta_0). \tag{A4}$$

The integration constant c can be found from the boundary condition that at $\zeta = 1$, $z = 0$. We note that as $\zeta_0 \to 1$, c becomes infinite and the free streamline moves towards $z = +\infty$.

Note added in proof: The two-dimensional solution described by equations (11) and (12) is also given, in a different conext, in Tuck & Dixon (1989) and used in Korobkin (1995) as a local solution.

REFERENCES

BENJAMIN, T. B. 1968 Gravity currents and related phenomena. *J. Fluid Mech.* **31**, 209–248.

COOKER, M. J. & PEREGRINE, D. H. 1990*a* Computations of violent motion due to waves breaking against a wall. *Proc. 22nd Intl Conf. Coastal Engng, Delft. ASCE*, pp. 164–176.

COOKER, M. J. & PEREGRINE, D. H. 1990*b* A model for breaking wave impact pressures. *Proc. 22nd Intl Conf. Coastal Engng, Delft. ASCE*, pp. 1473–1486.

COOKER, M. J. & PEREGRINE, D. H. 1992 Wave impact pressure and its effect upon bodies lying on the bed. *Coastal Engng* **18**, 205–229.

JOUKOWSKI, N. 1900 Über den hydraulischer Stoss in Wasserlietungsrohren. *Mem. Acad. Imperiale des Sciences de St. Petersburg* (transl. (1904) Waterhammer, *Proc. Am. Water Works Assoc.* **24**, 341–424).

KOROBIN, A. 1995 Impact of two bodies one of which is covered by a thin layer of liquid. *J. Fluid Mech.* **300**, 43–58.

TUCK, E. O. & DIXON, A. 1989 Surf-skimmer planing hydrodynamics. *J. Fluid Mech.* **205**, 581–592.